Consistent filtering for the lattice Boltzmann computation of aerodynamic noise.

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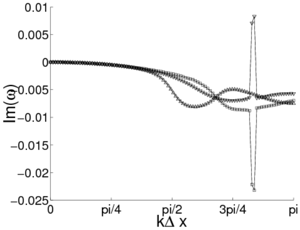
Introduction

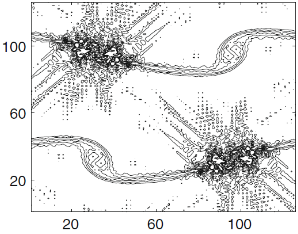
- BGK Lattice Boltzmann Method is known to be a low-dissipative scheme (Marié, Ricot, and Sagaut 2009[1]).
- Consequently, numerical stability is rarely provided for large Reynolds numbers.
- Numerous methods have been proposed to increase numerical stability (MRT, artificial viscosity, entropic formulation...).
- Lots of them induce a global over-dissipation and could damp some low-amplitude physical modes (acoustics).
- Stabilization procedure can drastically increase computational cost.

[1] S. Marié, D. Ricot, and P. Sagaut (2009). "Comparison between Lattice Boltzmann Method and Navier-Stokes high order schemes for Computational Aeroacoustics." In: *Journal of Computational Physics.* 228 (4), pp. 1056–1070

Introduction

- Brogi et al. 2017[2] introduce a regularisation step after collision to damp unphysical modes and manage to keep low enough dissipation of acoustic waves.
- Gendre et al. 2017[3] propose to use a two-relaxation time with enhanced interpolation for transition of resolution.
- Here some adaptive filtering technique consistent with LBM stencil and acoustic propagation is proposed for BGK and MRT models.





[2] F. Brogi et al. (2017). "Hermite regularization of the lattice Boltzmann method for open source computational aeroacoustics". In: *The Journal of the Acoustical Society of America* 142.4, pp. 2332–2345

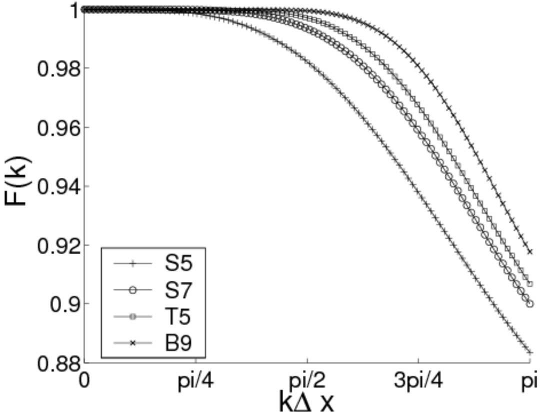
[3] Félix Gendre et al. (2017). "Grid refinement for aeroacoustics in the lattice Boltzmann method: A directional splitting approach". In: *Phys. Rev. E* 96 (2), p. 023311

1. Numerical models
   - Basic idea of adaptive filtering
   - BGK implementation
   - MRT implementation
2. Aerodynamic validation
   - 3D Taylor-Green-Vortex
   - Taylor-Green results
3. Acoustic Validation
   - Sound radiated by a square cylinder
   - Preliminary results
4. Conclusion and perspectives

Basic spatial filtering of a given quantity Q is performed as follow:

$$\langle Q(\mathbf{x})\rangle = Q(\mathbf{x}) - \sigma \sum_{j=1}^{D} \sum_{n=-N}^{N} d_n Q(\mathbf{x} + n\Delta x_j) \quad (1)$$

$0 < \sigma = Cste < 1$ and coefficients d_n depending on the filter stencil.



Adaptive filtering:

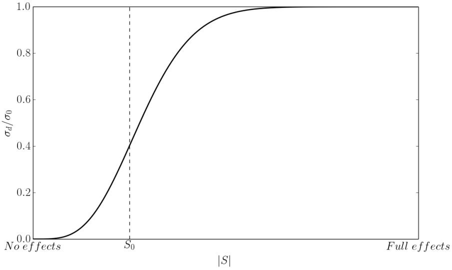
The filter coefficient σ depends on the shear stress amount:

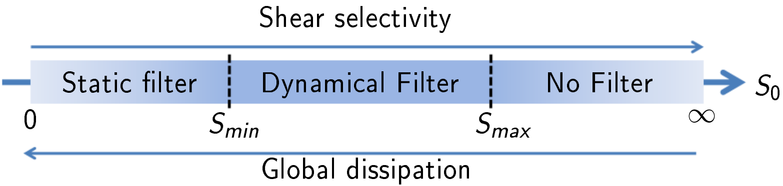
$$\sigma_a(\mathbf{x}) = \sigma_0 \xi(|S|) \tag{2}$$

$$\xi(|S|) = \left(1 - e^{-(|S(\mathbf{x})|/S_0)^2}\right)^2 \tag{3}$$

$|S| = \sqrt{2S_{ij}S_{ij}}$, S_0 is a threshold to be determined.

$$\sigma_a(\mathbf{x}) = \sigma_0 \left(1 - e^{-(|S|/S_0)^2}\right)^2$$





$$\sigma_d(\mathbf{x}) = \sigma_0 \left(1 - e^{-(|S|/S_0)^2}\right)^2$$

Threshold estimation:

S_0 could be linked with the maximum amount of shear stress S_{max}:
$S_0 = \epsilon S_{max}$

- S_{max} imposed to a constant.
- S_{max} computed from mean field.
- S_{max} based on g_α positivity.

LBM shear computation:

From a LBM framework, the shear stress can be obtained with:

$$2\rho\nu S_{ij} = -\sum_{\alpha} c_{\alpha,i} c_{\alpha,j} (g_\alpha - g_\alpha^{eq})) \quad (4)$$

Assuming the positivity of g_α and $\sum_{\alpha} c_{\alpha,i} c_{\alpha,j} g_\alpha^{eq} = \rho u_i u_j$

$$|S| = \frac{\mathbf{Q_f}}{2\rho\nu} = \frac{\sqrt{2\sum_\alpha c_{\alpha,i} c_{\alpha,j} g_\alpha^{eq} \sum_\alpha c_{\alpha,i} c_{\alpha,j} g_\alpha^{eq}}}{2\rho\nu} \leq \frac{\sqrt{2} u_i u_j}{2\nu}$$

Then:

$$S_{max} = \frac{\sqrt{2} U_0^2}{2\nu} \sim \frac{Re_\delta U_0}{\delta}$$

Lattice Boltzmann Method

Lattice Boltzmann algorithm:

$$g_\alpha^{coll}(\mathbf{x},t) = g_\alpha(\mathbf{x},t) - \frac{1}{\tau_g}\left[g_\alpha(\mathbf{x},t) - g_\alpha^{eq}(\mathbf{x},t)\right]$$

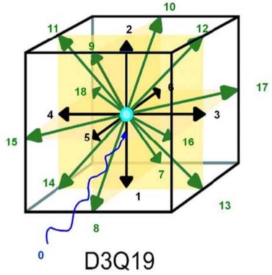
$$g_\alpha(\mathbf{x},t) = g_\alpha^{coll}(\mathbf{x} - \mathbf{c}_\alpha \Delta t, t - \Delta t)$$

$$\rho = \sum_\alpha f_\alpha$$

$$\rho\mathbf{u} = \sum_\alpha \mathbf{c}_\alpha f_\alpha$$

$$g_\alpha^{eq}(\mathbf{x},t) = \rho\omega_\alpha\Delta\left(1 + \frac{\mathbf{u}.\mathbf{c}_\alpha}{\tilde{c}_0^2} + \frac{(\mathbf{u}.\mathbf{c}_\alpha)^2}{2\tilde{c}_0^4} - \frac{|\mathbf{u}|^2}{2\tilde{c}_0^2}\right)$$

LBM recovers compressible Navier-Stokes equations in the limit of low Mach Numbers $\mathcal{O}(M^3)$



$$p = \tilde{c}_0^2\rho$$
$$\tilde{c}_0^2 = 1/3$$
$$\omega_0 = 1/3$$
$$\omega_{1-6} = 1/18$$
$$\omega_{7-18} = 1/36$$

Choosing the filtered quantity

3 different possibilities with increasing computational cost:

1. Filtering momenta: ρ, u_x, u_y, u_z (4 tables).
2. Filtering distribution functions: g_α (19 tables).
3. Filtering collision operator: $-\frac{1}{\tau_g}(g_\alpha - g_\alpha^{eq})$ (19 tables).

Then the overall algorithm becomes:

Modified algorithm for moments filtering:

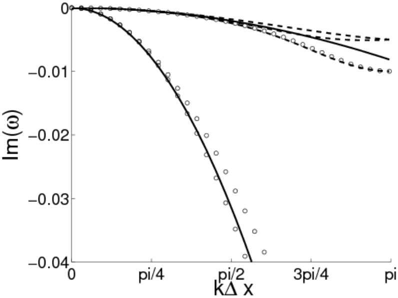
- Estimation of S_0.
- Compute $|S|$ with (4) and update σ_d
- Collision Step
- (Collision Filtering)
- Propagation Step
- Update moments
- (Moments filtering)
- Update g_α^{eq}

Towards adaptive relaxation times

MRT Collision

$$\mathbf{m}_\alpha^* = \mathbf{m}_\alpha - S\left(\mathbf{m}_\alpha - \mathbf{m}_\alpha^{eq}\right)$$
$$g_\alpha(\mathbf{x} + \mathbf{c}_\alpha \Delta t, t + \Delta t) = M^{-1}\mathbf{m}_\alpha^*(\mathbf{x}, t)$$

In the MRT framework, the filtering step is not needed.



$$(m_1, \cdots, m_9) = (\rho, e, \epsilon, \rho u_x, q_x, \rho u_y, q_y, p_{xx}, p_{xy})$$

The function $\xi(|S|)$ is used to switch from MRT in sheared region to BGK in uniform flow region:

$$S' = diag\left(0, s_e', s_\epsilon', 0, s_q', 0, s_q', s_\nu, s_\nu\right) \quad (5)$$

where $s' = s + (1 - \xi)(s_\nu - s)$

Then the overall algorithm becomes:

Modified algorithm for adaptive relaxation times:

- Compute $|S|$ with (4) and update ξ
- Update relaxation time matrix S'
- Collision Step in the momentum space
- Propagation Step
- Update moments
- Update equilibrium

Initialization of the macroscopic variables

The initialization of the Taylor-Green vortex is done by setting velocity and pressure in a cubic and fully periodic domain of size 2π:

$$\begin{cases} p = p_\infty + \dfrac{\rho_\infty U_\infty^2}{16}[\cos(2z)+2][\cos(2x)+\cos(2y)] \\ u = U_\infty \sin(x)\cos(y)\cos(z) \\ v = -U_\infty \cos(x)\sin(y)\cos(z) \\ w = 0 \end{cases} \tag{6}$$

Parameters:

- Imposing $Re = 1600$, $M_\infty = 0.085$, $\rho_\infty = 1$, $dx = 2\pi/n_x$
- Induces: $U_\infty = 0.049$, $p_\infty = 1/3$, $\tau_g = 0.5 + 1.46 n_x 10^{-5}$

Initialization of the distribution functions

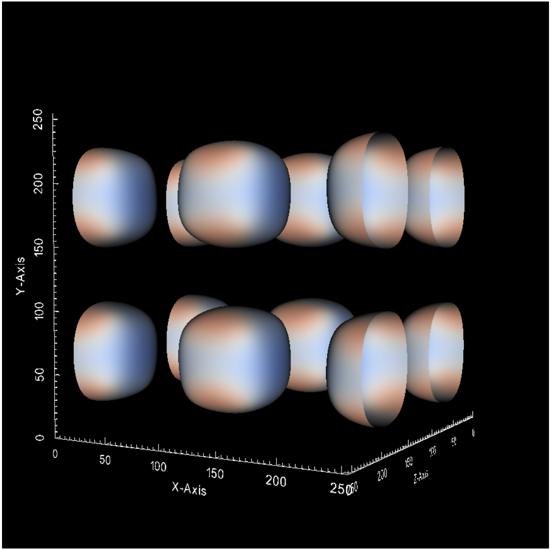
In order to avoid spurious oscillations from initialization, the distribution functions are initialized with their non-equilibrium part:

Initialisation procedure from Skordos 1993[4]

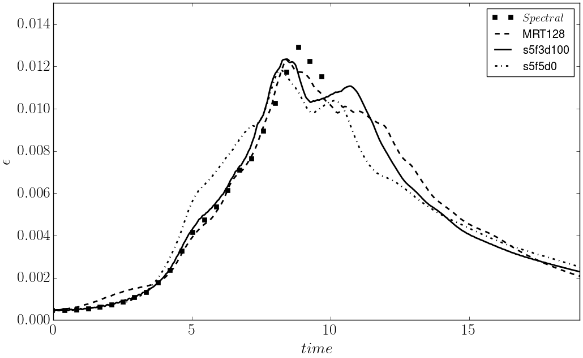
$$
\begin{aligned}
g_\alpha^{init} &= g_\alpha^{eq} + g_\alpha^{neq} \\
g_\alpha^{neq} &= -\frac{\omega_\alpha \tau_g}{\tilde{c_0}^2}\left[\left(c_{\alpha,i}c_{\alpha,j} - \tilde{c_0}^2\delta_{ij}\right)\frac{\partial \rho u_i}{\partial x_j}\right]
\end{aligned}
$$

Gradients are evaluated with a centred 2^{nd} order finite difference scheme.

[4] P. Skordos (1993). "Initial and Boundary Conditions for the lattice Boltzmann Method." In: *Physical Review E* 48.6, pp. 4823–4842


Y-Axis
0
50
100
150
200
250
X-Axis
Z-Axis

Dissipation rate of kinetic energy



Comparison between Adaptive and classical techniques.

- Results on a 128^3 grid
- Adaptive filtering give similar results than the MRT model.
- Adaptive filtering with 3-point stencil gives better results than static 5-point.
- Mode details in Marié and Gloerfelt 2017[5]

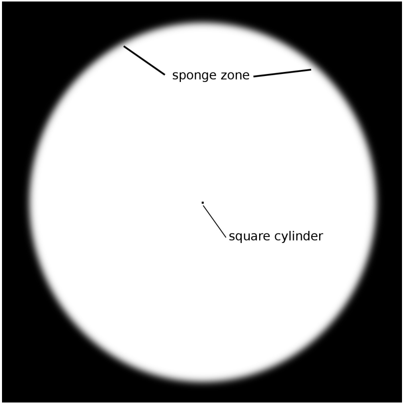
[5] S. Marié and X. Gloerfelt (2017). "Adaptive filtering for the lattice Boltzmann method." In: *Journal of Computational Physics* 333C, pp. 212–226

Case setup

Parameters

- Reynolds $R_D = 150$
- Mach number $M_a = 0.3$
- the square size is $D = 5$
- the domain size is $300D \times 300D$
- The domain is initialized with a uniform flow and periodic boundary conditions.
- A circular sponge zone is defined at the domain boundary to damp the outgoing structures.

Case setup



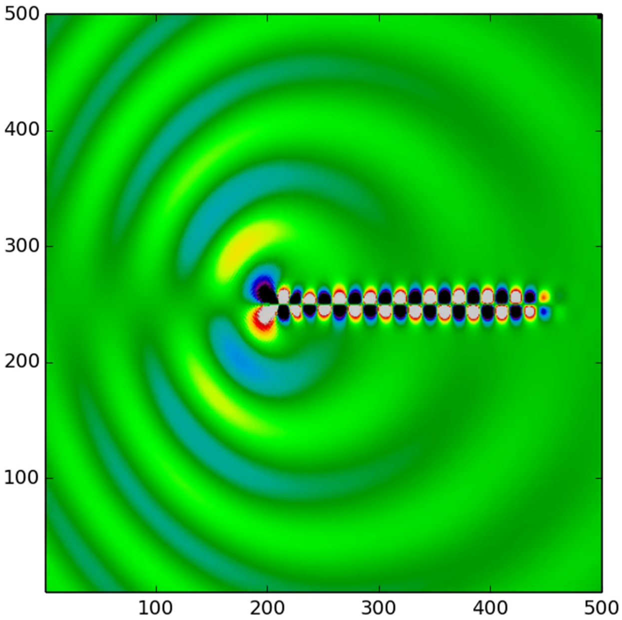
- Details about LBM sponge zone from Xu and Sagaut 2011[6].
- Here the filter coefficient σ and the relaxation time τ_ν are set to high values in the sponge zone.
- For ART model, the switcher is set to 1 in the sponge zone.

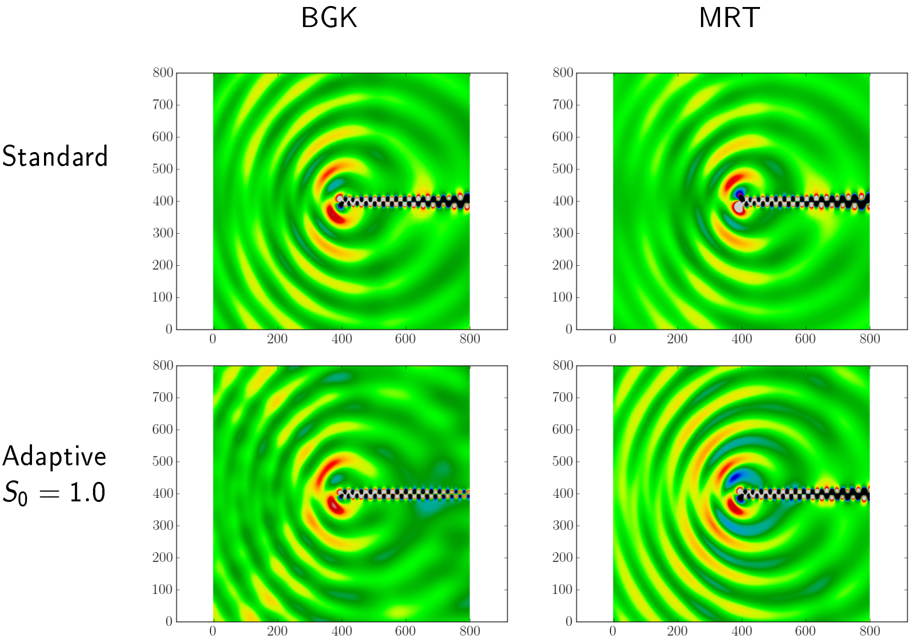
[6] Hui Xu and Pierre Sagaut (2011). "Optimal low-dispersion low-dissipation LBM schemes for computational aeroacoustics". In: *Journal of Computational Physics* 230.13, pp. 5353 –5382.

The von-Karmann instability is known to generate sound-waves at a given characteristic frequency: $S_t = \dfrac{f.D}{U_\infty} \sim 0.17$

About resolutions

- Aerodynamic resolution : D
- Acoustic resolution : $\lambda = \dfrac{D}{S_t.M_\infty} \sim 20D$
- Difficult to handle this resolution in the whole domain.
- Consequently,far field acoustic is often a coarse region where numerical dissipation is high.
- Here, only uniform resolution is considered.

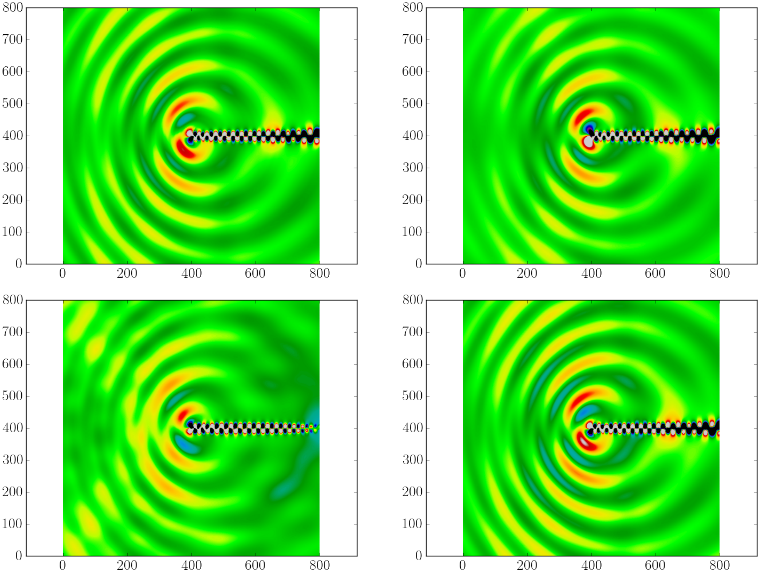

500
400
300
200
100
100
200
300
400
500


BGK
MRT
Standard
Adaptive
$S_0 = 1.0$
800
700
600
500
400
300
200
100
0
0
200
400
600
800

BGK MRT

Standard

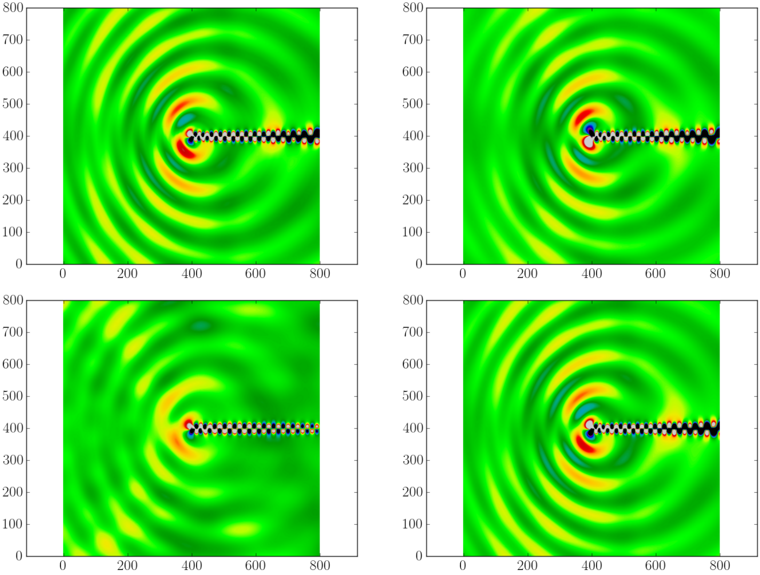
Adaptive $S_0 = 0.5$



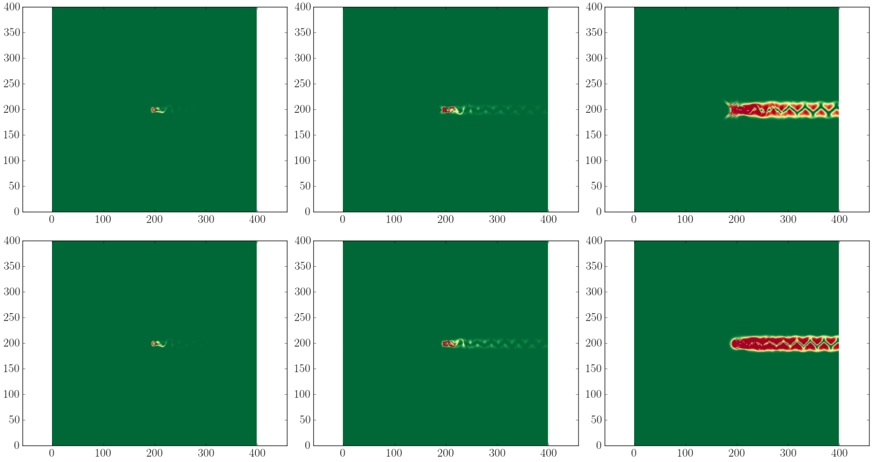
BGK | MRT

Standard

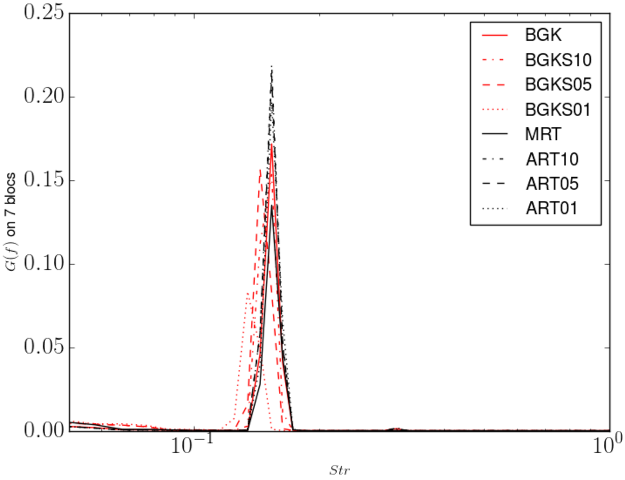
Adaptive
$S_0 = 0.1$



Maps of $\xi(|S|)$



PSD of the acoustic pressure at a $80D$ distance from the square.



Conclusion

- The use of adaptive filters with BGK operator give similar results than the MRT model on a 3D TGV dynamics.
- Adaptivity allows to use low stencil filters, consistent with LBM models.
- Adaptivity allows to keep aerodynamic accuracy and keeps the acoustic waves free of numerical dissipation.
- Introducing an adaptive filtering induces an early development of the von Karmann instability.
- Adaptive relaxation times gives promising results for high Reynolds acoustic generation and propagation.

Perspectives

- Need for broad-band noise validation to get a wide range of acoustic resolutions. (ie. 3D turbulent jet)
- Quantitative comparison to other techniques (Regularization, TRT...)
- Focus on the switcher shape (Lighthill source term sensitive switcher)
- Take the wall distance into account explicitly (Boundary Layer correction,van Driest...)
- Apply adaptive relaxation times approach to a larger range of physical application (multiphase, large scales multicomponent flow...)

Thanks for your attention