On the use of adaptive filtering for multiphase Lattice Boltzmann Model

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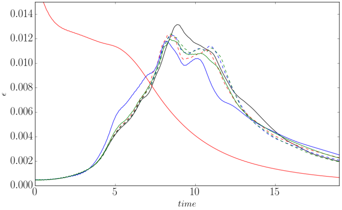
Introduction

- BGK numerical stability is rarely provided for large Reynolds numbers.
- Numerous methods have been proposed to increase numerical stability (MRT, entropic formulation, Cumulant...).
- Lots of them induce a global over-dissipation and could damp some needed physical modes (acoustics).
- LBM instabilities are known to be high frequencies.
- Ricot et al. 2009[1] proposed to use selective filters to damp only high wavenumbers instabilities.
- High order filters are then required to ensure accurate selectivity thus increasing numerical stencil.
- Need for an enhanced filtering strategy with physical selectivity.

[1] D. Ricot et al. (2009). "Lattice Boltzmann Method with selective viscosity filters". In: *Journal of Computational Physics*. 228.12, pp. 4478–4490

- Marié and Gloerfelt 2016[2] recently proposed to use adaptive filters acting only in sheared region.
- Validated on 3D Taylor-Green Vortex with low-stencil filters.
- Can we apply such filters with multiphase models ?



- Multiphase Lattice-Boltzmann models have different nature: interparticle interaction, nearly-incompressible...
- The mean-field or nearly incompressible models (such as He,Chen,Zhang) are adapted to dense gas and RT-type instability but have stability issues for large density ratios.
- Extended versions of HCZ model exists in the litterature with larger computational cost.

[2] S. Marié and X. Gloerfelt (2016). "Adaptive filtering for the lattice Boltzmann method". In: *Submitted to Journal of Computational Physics*

1. Numerical models
   - The HCZ model
   - Adaptive filtering
2. Application to the Rayleigh-Taylor instability
   - Test case implementation
   - Comparison with classical NS-Scheme
3. Conclusion and perspectives

He, Chen, and Zhang 1999[3]) model

BGK on a D2Q9 lattice with force terms:

$$f_\alpha^+ = \mathrm{BGK}(f_\alpha) - \frac{2\tau-1}{2\tau}(\mathbf{c}_\alpha - \mathbf{u}).\frac{\nabla\psi(\phi)}{RT}\Gamma_\alpha(\mathbf{u})$$

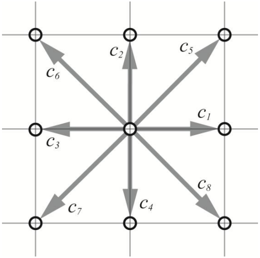
$$g_\alpha^+ = \mathrm{BGK}(g_\alpha) - \frac{2\tau-1}{2\tau}(\mathbf{c}_\alpha - \mathbf{u}).\left[\nabla\psi(\rho)(\Gamma_\alpha(\mathbf{u}) - \Gamma_\alpha(0)) - \Gamma_\alpha(\mathbf{u})(\mathbf{F_s} + \mathbf{G})\right]$$

Equilibrium state:

$$\Gamma_\alpha(\mathbf{u}) = \omega_\alpha\left(1 + \frac{\mathbf{u}.\mathbf{c}_\alpha}{RT} + \frac{(\mathbf{u}.\mathbf{c}_\alpha)^2}{2(RT)^2} - \frac{|\mathbf{u}|^2}{2RT}\right)$$

$$f_\alpha^{eq} = \phi\Gamma_\alpha(\mathbf{u})$$

$$g_\alpha^{eq} = \omega_\alpha\left[p + \rho RT\left(\frac{\mathbf{u}.\mathbf{c}_\alpha}{RT} + \frac{(\mathbf{u}.\mathbf{c}_\alpha)^2}{2(RT)^2} - \frac{|\mathbf{u}|^2}{2RT}\right)\right]$$



[3] X. He, S. Chen, and R Zhang (1999). "A Lattice Boltzmann Scheme for Incompressible Multiphase Flow and Its Application in Simulation of RayleighâTaylor Instability". In: *Journal of Computational Physics* 152, pp. 642–663.

He-Chen-Zhang model

Moments:

$$\begin{aligned} \phi &= \sum_\alpha f_\alpha \\ p_h &= \sum_\alpha g_\alpha - \frac{1}{2}\mathbf{u}.\nabla\psi(\rho) \\ \rho RT\mathbf{u} &= \sum_\alpha \mathbf{c}_\alpha g_\alpha + \frac{RT}{2}(\mathbf{F_s} + \mathbf{G}) \end{aligned}$$

Thermodynamics:

$$\begin{aligned} \psi(\rho) &= p_h - \rho RT \\ \psi(\phi) &= p_{th} - \phi RT \end{aligned}$$

Carnahan-Starling equation of state:

$$p_{th} = \phi RT \frac{1 + \phi + \phi^2 - \phi^3}{(1-\phi)^3} - a\phi^2$$

Selective filtering of a given quantity Q is performed as follow:

$$\langle Q(\mathbf{x}) \rangle = Q(\mathbf{x}) - \sigma \sum_{j=1}^{D} \sum_{n=-N}^{N} d_n Q(\mathbf{x} + n\Delta x_j) \quad (1)$$

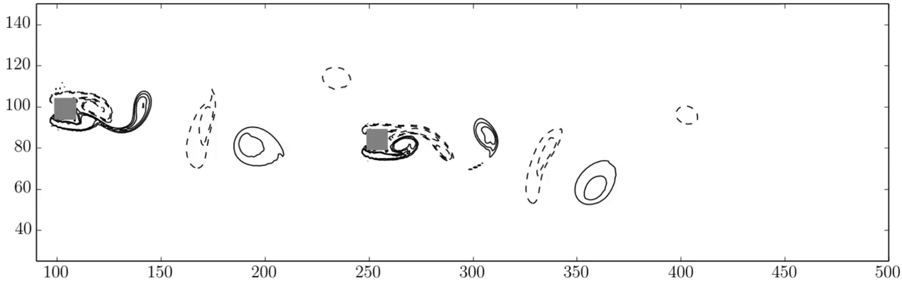
$0 < \sigma = Cste < 1$ and coefficients d_n depending on the filter order.

Adaptive filters for turbulence:

The filter coefficient σ is sensitive to the shear stress amount:

$$\sigma = \sigma_d(\mathbf{x}) = \sigma_0 \left(1 - e^{-(|S|/S_0)^2}\right)^2 \quad (2)$$

$|S|$ can be estimated with $\sum_\alpha c_{\alpha,i} c_{\alpha,j} (g_\alpha - g_\alpha^{eq})$,
S_0 is a shear sensitivity threshold based on stability criteria.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Adaptive filtering

Sensitive quantity for multiphase models ?:

$$f_\alpha^+ = \mathbf{BGK}(f_\alpha) - \frac{2\tau - 1}{2\tau}(\mathbf{c}_\alpha - \mathbf{u}).\frac{\nabla\psi(\phi)}{RT}\Gamma_\alpha(\mathbf{u})$$

Multiphase filters:

Coefficient σ depends on density gradient:

$$\sigma = \sigma_d(\mathbf{x}) = \left(1 - e^{-(\nabla\psi/\psi_0)^2}\right)^2 \quad (3)$$

ψ_0 is the sensitivity threshold $\sim \dfrac{\rho_h - \rho_l}{\eta}$.

η is an imposed reference length in lattice unit.

No additional computational time:

$\nabla\psi$ already computed from the source terms in the overall algorithm:

$$\nabla\psi = \sum_{\alpha\neq 0} \frac{\omega_\alpha \mathbf{c}_\alpha.\mathbf{i}[\psi(\mathbf{x}+\mathbf{c}_\alpha) - \psi(\mathbf{x}-\mathbf{c}_\alpha)]}{2c_0^2} \quad (4)$$

Filtering of the index function ϕ

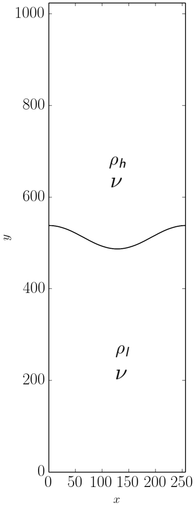
Then the overall algorithm becomes:

- Compute $\nabla\psi(\rho)$ with (4) and update σ_d
- Collision Steps
- Propagation Steps
- Update ϕ and moments
- Filtering of ϕ
- Update g_α^{eq} and f_α^{eq}

Single-mode perturbation

The Rayleigh-Taylor instability is initialized with a 10% perturbation:

- $y_h = 0.1 W \cos(2\pi x / W)$
- $A_t = \dfrac{\rho_h - \rho_l}{\rho_h + \rho_l}$
- $Re = \dfrac{\sqrt{Wg}\, W}{\nu}$
- $\sqrt{Wg} = 0.04$
- Time scale: $T = \sqrt{W/g}$
- 256×1024 grid



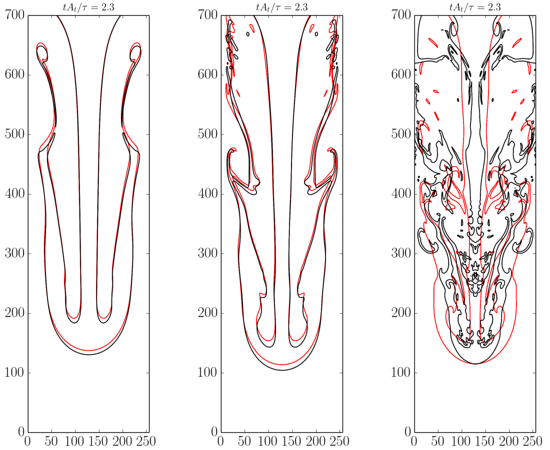
TVD Scheme

- Scheme used in Mongruel et al. 2009[4]
- compressible 2nd order TVD scheme.
- Acoustic-convection splitting.
- Super-Bee type limiter for convection step.
- Isentropic equation of state.
- Computations are made on the same grid with the same parameters and same $CFL = 0.57$.

[4] A. Mongruel et al. (2009). "Early post-impact time dynamics of viscous drops onto a solid dry surface". In: *Physics of Fluids* 21.3

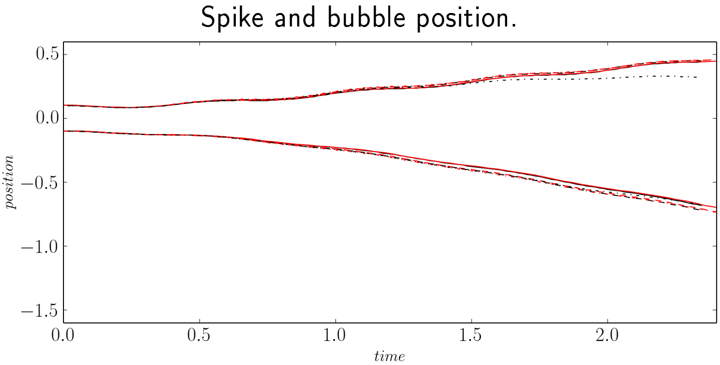
$A_t = 0.5$

$\eta = 0$ (No-filter) −−LBM −−NS



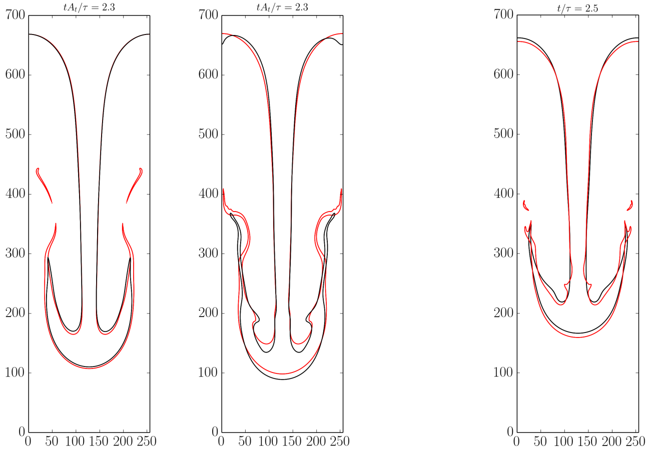
$Re = 256$ $Re = 1024$ $Re = 4096$

$A_t = 0.5$



$-Re = 256, - - Re = 1024, -.Re = 4096$

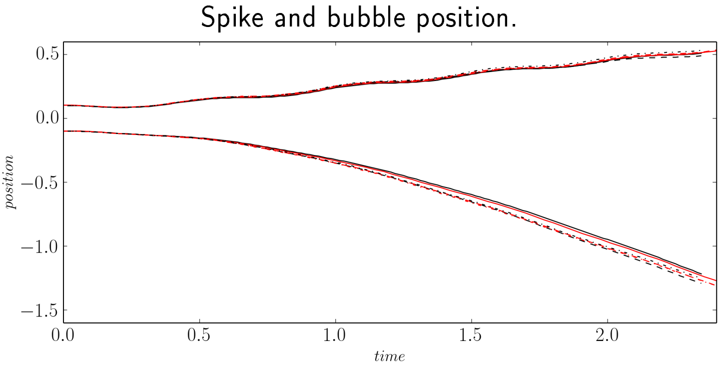
$A_t = 0.8 \qquad \eta = 2 \qquad$ – –LBM – –NS



$Re = 256 \qquad Re = 1024 \qquad Re = 2048$(Chao et al. 2011[5])

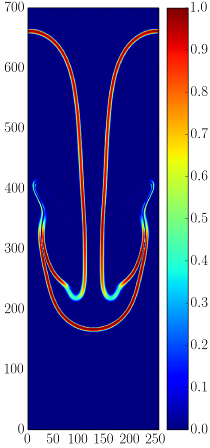
[5] Jianghui Chao et al. (2011). "A filter-based, mass-conserving lattice Boltzmann method for immiscible multiphase flows". In: *International Journal for Numerical Methods in Fluids* 66.5, pp. 622–647

$A_t = 0.8$

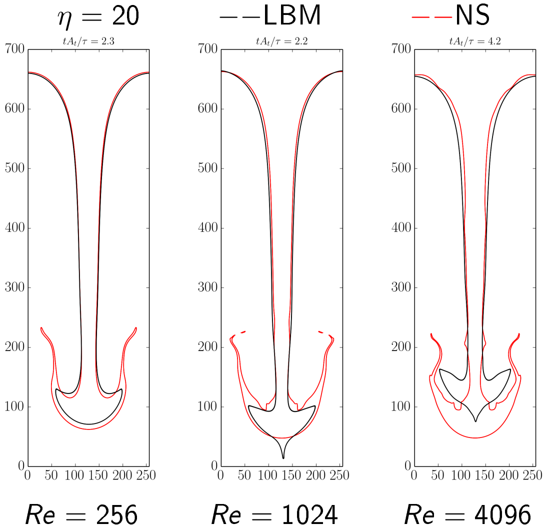


$-Re = 256$, $- - Re = 1024$, $-.Re = 4096$

Distribution of σ



$A_t = 0.9$


$\eta = 20$
−−LBM
−−NS
$tA_t/\tau = 2.3$
$tA_t/\tau = 2.2$
$tA_t/\tau = 4.2$
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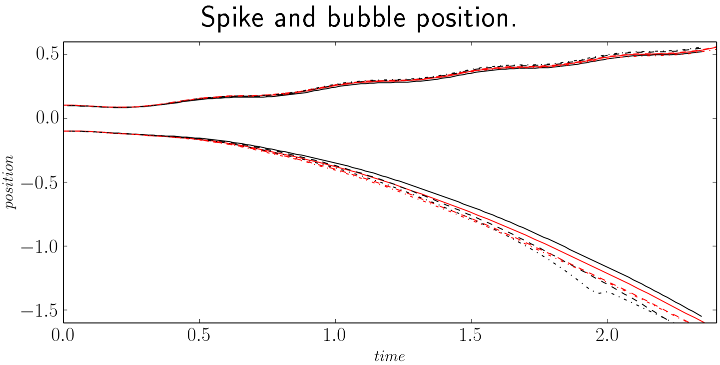
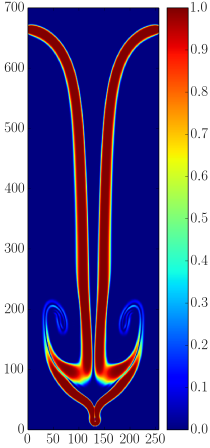
0 50 100 150 200 250
$Re = 256$
$Re = 1024$
$Re = 4096$

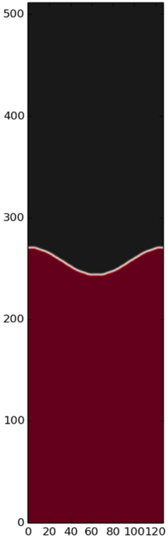
$A_t = 0.9$



$-Re = 256, \; -- \; Re = 1024, \; -.Re = 4096$

Distribution of σ

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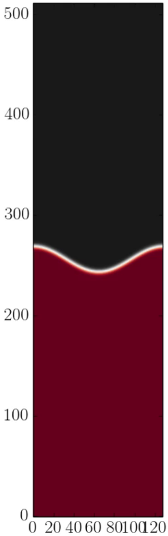
$A_t = 0.5$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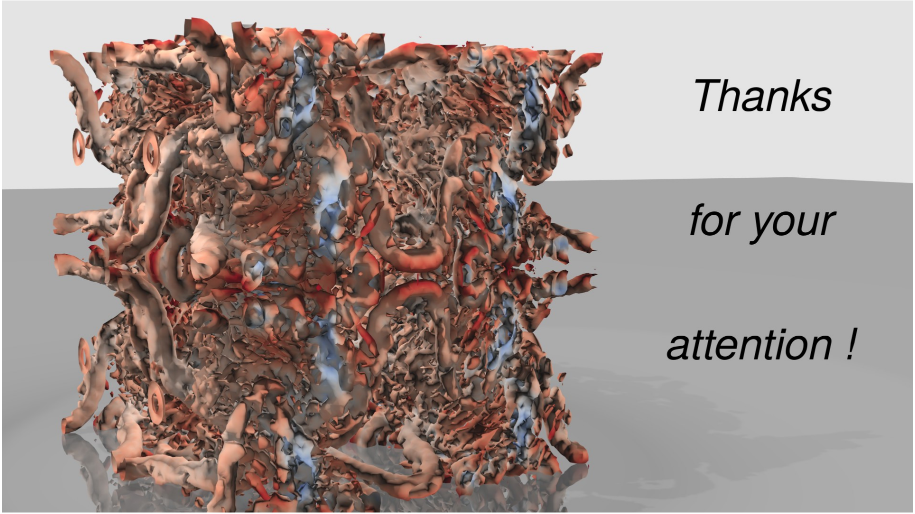
$A_t = 0.9$

Summary:

- The use of selective spatial filters with a coefficient based on $\nabla\psi$ allows low-stencil filters with multiphase flows.
- Adaptive filters can improve stability for moderate density ratio without modifying underlying dynamics.
- For large density ratio, stability is ensured but interface is scattered for high Reynolds numbers.

Future work:

- Apply adaptive filters on the other kind of multiphase flow.
- Perform a linear stability analysis of the multiphase models.
- Apply it on 3D parallel simulations (PhD. of L.Vienne to begin in October)
- Adaptive filters can be applied on the growing-up family of the (low-stencil) lattice Boltzmann models.


Thanks
for your
attention !