A STUDY OF SUBGRID MODELS IN LATTICE-BOLTZMANN-BASED LARGE EDDY SIMULATION

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Summary Despite there is an exponentially growing literature devoted to the LBM, the progress of LBM in studying turbulent flow is not fully satisfactory compared with its achievements in other aspects at present time. In this paper, we propose to extend the mapping technique to recover an IR consistent Smagorinsky model[1] to the LBM framework. The decaying isotropic turbulence (DHIT) is selected as a test case. The objectives are to investigate the sensitivity of turbulence evolution upon the various Smagorinsky SGS models, the model coefficients and the grid resolution, and to validate the efficiency and the accuracy of LES-LBM.

MATHEMATICAL FORMULATIONS

The filtered lattice Boltzmann equation for LES is solved using single relaxation time approximation following Bhatnagar, Gross and Krook (BGK)[2] as

$$\overline{f}_\alpha\left(\boldsymbol{x}+\boldsymbol{e}_\alpha\delta t,\ \ t+\delta t\right)-\overline{f}_\alpha\left(\boldsymbol{x},t\right)=-\frac{1}{\tau_w}\left[\overline{f}_\alpha\left(\boldsymbol{x},t\right)-\overline{f}_\alpha^{(e)}\left(\boldsymbol{x},t\right)\right] \qquad (1)$$

Here, $f_\alpha\left(\boldsymbol{x},t\right)$ is the distribution function at a node $\boldsymbol{x}$ and time t with particle velocity vector $\boldsymbol{e}_\alpha$, and τ is relaxation time. $f_\alpha^{(e)}\left(\boldsymbol{x},t\right)$ in Eq. (1) is local equilibrium distribution function at each node: A cubic lattice model D3Q19 is used here to simulate the homogenous isotropic turbulence. The mass density ρ and macroscopic local velocity $\boldsymbol{u}$ are defined in term of the particle distribution function. The kinematic viscosity ν depend on the lattice relaxation time.

In LES-LBM, to implement IR consistent Smagorinsky model and the classical Smagorinsky model, an additive space and time variable relaxation time scale τ_τ is introduced into the effective relaxation time τ_w and $\tau_w{}^*$, respectively.

$$\tau_w=\frac{1}{2}+\frac{1}{c_s{}^2\delta t}\left[\nu_0+C^2\Delta_L^2\left|\overline{S}\right|\right] \qquad (2a)$$

$$\tau_w^*=\frac{1}{2}+\frac{3}{c^2\delta t}\sqrt{\left(C_\infty\Delta/\gamma\right)^4\left|\overline{S}\right|+\nu_0{}^2} \qquad (2b)$$

The total effective relaxation time τ_w used in filtered LBE is calculated from Eq. (2a,b) once the Smagorinsky constant C, the lattice length unit Δ_L and the kinematic viscosity ν_0 are given.

RESULTS AND DISCUSSION

We perform LES-LBM of decaying isotropic turbulence with both the classical Lilly-Smagorinsky model (results being denoted with "Lilly-1"(Cs=0.10) and Lilly-2"(Cs=0.18) in figures) and the IR consistent Smagorinsky model (denoted by "M&S-1" (Cs=0.10) and "M&S-2" (Cs=0.18) in figures). We also perform DNS-LBM to enable accurate comparisons.

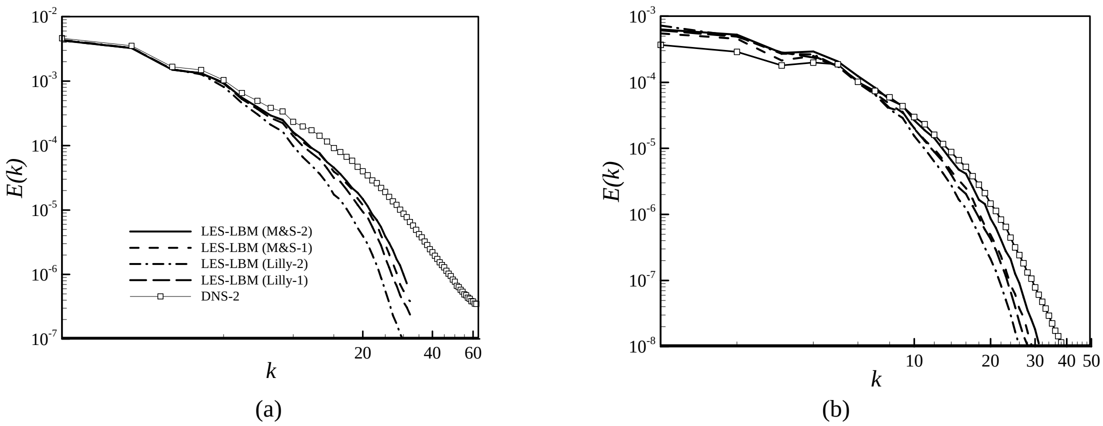
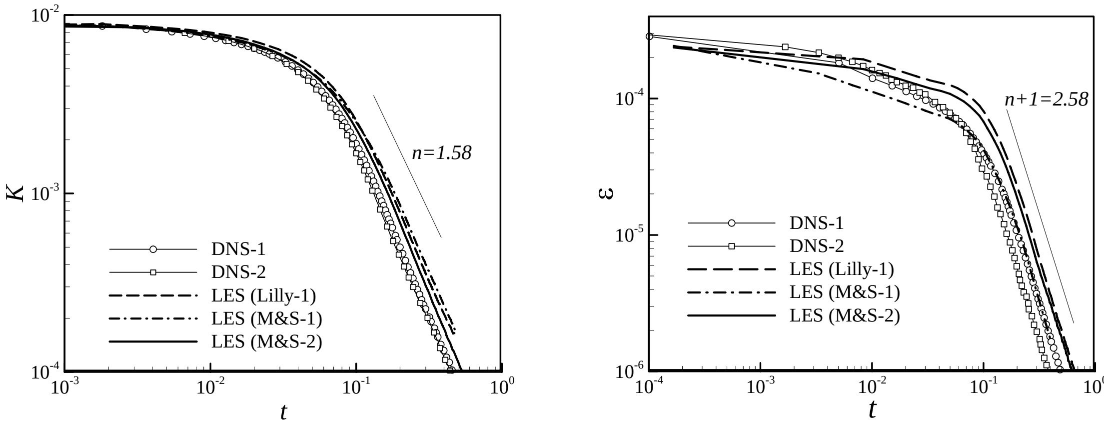


Fig.1. The instantaneous three-dimensional energy spectra at (a) t =0.05 and (b) t=0.18 obtained from DNS-LBM (192^3) and LES-LBM with different SGS models (Lilly and M&S).

Figure 1 displays the three-dimensional instantaneous energy spectra with different LES-LBM cases. The LES spectra are compared against DNS spectrum at the same time. The IR consistent model performs better than the other models in the different time. It also yields a better prediction of time evolution of the statistical quantities.

Figure 2a and Figure 2b show the time evolution of total turbulent kinetic energy and the dissipation rate given by three LES-LBM cases and DNS-LBM. It is well known that the energy decay exponent is closely related to the low wavenumber portion of the three-dimensional spectrum, and is affected by many features of the initial spectrum as well as Re_λ. Saffman[3] suggested that for DHIT the low Reynolds number exponent was shown to be 3/2, and the high Reynolds number exponent limit to be 6/5 which is commonly observed in experiments. Batchelor and Townsend[4] presented the first analysis and experiments for very low Reynolds number decaying turbulence and suggested that the exponent should be 5/2 in the final period of decay.



(a) (b)

Fig.2a. The total turbulent kinetic energy at different resolutions using LES-LBM and DNS-LBM. Fig.2b. The dissipation rate at different resolutions using LES-LBM and DNS-LBM.

The decay exponent law from present LES simulations is $n \approx 1.58$, which is agree well with the both DNS results and is close to the value of Djenidi's DNS[5] ($n \approx 1.53$). Our results also close to Lavoie's measurements[6] which the total kinetic energy is proportional to $t^{-1.5}$. Figure 2b shows that the collapse of ε predicted by both Smagorinsky models is really slower than the DNS value. The phenomenon corresponds to the evolution of energy spectrum in Fig. 1. The IR consistent models yield the results closer to the DNS ones. The decay exponent $n+1$ is estimated to be 2.58 in our simulations

CONCLUSIONS

In the present paper, we extend the study of the decaying isotropic turbulence with large eddy simulation based on lattice Boltzmann Equation, and investigated the performance of the standard Smagorinsky model and the IR consistent Smagorinsky model. The results are assessed via comparisons with the theory and the experimental data as well as DNS data. A very encouraging result is that the well known decay exponents of the kinetic energy and the dissipation rate are reproduced. Other results are found to be consistent with simulations made by different numerical schemes and measurements of grid turbulence.

Overall, the present study provides detailed numerical data and analysis against which such various subgrid-viscosity SGS models can be tested in the frame of LBM. We recall that LES-LBM is a potentially viable tool for study of turbulent flows and should be paid more attention to develop reasonable turbulence models.

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References

[1] J. Meyers, P. Sagaut. "On the model coefficients for the standard and the variational multi-scale Smogorisky model," J. Fluid Mech. 569, 287-319 (2006).

[2] P.L. Bhatnagar, E.P. Gross, M. Krook, "A model for collision processes in gases. I. Small amplitude processes in charged and neutral one-component systems," Phys. Rev. 94 511–525 (1954).

[3] G. K. Batchelor, A. A. Townsend, "Decay of turbulence in the final period," Proceedings of the Royal Society of London Series A, 194, 527 (1948).

[4] P. G. Saffman, "The large scale structure of homogenous turbulence," J. Fluid Mech. 27, 581-593 (1967).

[5] L. Djenidi, "Lattice Boltzmann simulation of grid-generated turbulence," J. Fluid Mech. 552, 13-35 (2006).

[6] P. Lavoie, P. Burattini, L. Djenidi, R. A. Antonia, "Effect of initial conditions on decaying grid turbulence," Exps. Fluids. 39, 865-874 (2005).