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THE BRINKMAN PENALIZATION TECHNIQUE FOR POROUS-FLUID MEDIA: A LATTICE BOLTZMANN AND SEMI-LAGRANGIAN VORTEX METHOD COMPARISON.

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INTRODUCTION

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The presence of porous layers around solid bodies modifies the flow behavior at the solid-porous-fluid interface. In nature, porous media are known for their flow regularization properties (e.g. forest canopy, velvet-like feathers under bird wings, ...) and the use of porous coatings at the surface of solid obstacles immersed in a fluid flow has been employed across several areas to change the flow dynamics. Modeling flows in porous media therefore appears as a real issue to design and simulate passive flow control strategies. In lattice Boltzmann methods, the treatment of fluid flows in porous media can be carried out with different types of boundary conditions. In the present work, the Brinkman penalization method [1] is used. This approach models the porous medium by adding a forcing term including the porosity and the permeability of the medium. The Brinkman penalization holds for the whole domain, it is local and has a negligible computational cost. This strategy, recently used with LBM to model complex diffusion in porous media [2], has also been successively applied in the context of semi-Lagrangian vortex methods [5] for the discretization of incompressible Brinkman-Navier-Stokes equations. The present work therefore proposes a numerical comparison between two alternative and non-traditional methods, namely a mesoscopic approach (the lattice Boltzmann method, denoted LBM in the following) and a hybrid Eulerian-Lagrangian macroscopic one (the semi-Lagrangian vortex method, denoted VM in the following), in their ability and efficiency to simulate the flow around a porous body with the Brinkman penalization technique. As enhanced by the authors in [4], the LBM and VM methods belong to families of methods where the flow is discretized in a non-macroscopic way and where the notion of particles is a common aspect. They therefore show structural similarities and have been also shown in this study to present their own advantages, like the low-dissipative and low-dispersive properties of the VM and the high accuracy at fine grid resolutions and the convergence order of LBM. Further comparisons of these two approaches are here proposed in the context of porous flow simulations.

THE BRINKMAN PENALIZATION

The Brinkman penalization technique has been introduced to model the presence of a porous body [7]. The idea is to add a body force in the macroscopical equation set. This body force takes into account the Reynolds number and the Darcy number and writes. The adimensionalized formulation of the forcing term is given by:

$$\mathbf{F_p} = -\frac{\phi}{ReDa}\mathbf{u} \tag{1}$$

where, ϕ, the porosity of the immersed body, has to be close to 1 [8]. This macroscopic formulation takes different implementation strategies for Vortex method and Lattice Boltzmann Methods.

Implementation in the VM framework

In the VM context, we solve the incompressible Navier-Stokes equations in their adimensionalized velocity($\mathbf{u}$) - vorticity($\boldsymbol{\omega}$) formulation. Adding the Brinkman penalization forcing term in these equations gives the so-called penalized Vorticity-Transport-Equation:

$$\partial_t\boldsymbol{\omega} + (\mathbf{u}\cdot\nabla)\boldsymbol{\omega} - (\boldsymbol{\omega}\cdot\nabla)\mathbf{u} = \frac{1}{Re}\Delta\boldsymbol{\omega} + \nabla\times\mathbf{F_p} \tag{2}$$

$$\Delta\mathbf{u} = -\nabla\times\boldsymbol{\omega}, \tag{3}$$

Such equations allow to model the flow in the whole domain thanks to the dimensionless penalization coefficient $-\phi/ReDa$. Indeed, at a given Re number, varying the value of Da thus directly defines the different media: in the fluid, the permeability goes to infinity, thus the fluid can be considered as a porous media with infinite permeability meaning that the penalization term vanishes. For regions with finite values of Da, one models a porous medium in which the flow has a Darcy velocity.

In the present VM framework, the system of equations (2)-(3) is solved with a fractional step technique (where the diffusive, convective and stretching effects are handled successively within one time step) and by using a semi-Lagrangian approach where the convection of the vorticity field is performed in a Lagrangian way and all the other substeps are resolved on a grid using classical Eulerian schemes (FD, spectral) thanks to the regular remeshing of the Lagrangian particles on the grid. The resolution of the penalization equation $\partial_t\boldsymbol{\omega} = \nabla\times F_p$

is one of the fractional step of the global algorithm, and such equation is solved on the grid with an implicit Euler scheme for time integration and FD scheme for the discretization of the curl operator :

$$\boldsymbol{\omega}^{n+1} = \boldsymbol{\omega}^n + \nabla \times \left(\frac{-\lambda\chi\Delta t\ \mathbf{u}^n}{1+\lambda\chi\Delta t} \right) \tag{4}$$

Implementation in the LBM framework

The LBM model used for this comparison is based on the D3Q19 incompressible formulation of the Multiple Relaxation time [3]. The implementation of the MRT model is based on a collision step done in the momentum space:

$$\left\{ \begin{array}{lcl} \mathbf{m}^{coll} & = & \mathbf{m} - \mathcal{S}(\mathbf{m}-\mathbf{m}^{eq}) + (1-\frac{dt}{2})\mathcal{S}\mathcal{M}\mathbf{S} \\ \mathbf{g}(\mathbf{x},t) & = & \mathcal{M}^{-1}\mathbf{m}^{coll}(\mathbf{x}-\mathbf{c}_\alpha dt, t-dt) \end{array} \right. \tag{5}$$

where the matrix $\mathcal{M}$, transforms the distribution functions $\mathbf{g}$ into moments $\mathbf{m}$ and $\mathbf{m}^{eq}$ is computed from the standard second order equilibrium distribution function. The diagonal matrix $\mathcal{S}$ contains the relaxation rates associated to each moments to optimize numerical stability [3]. The source term $\mathbf{S}$ is computed according to the standard Guo formulation [6] and include the body force (1):

$$S_\alpha = \omega_\alpha \left[\frac{\mathbf{c}_\alpha - \mathbf{u}}{c_0^2} + \frac{(\mathbf{c}_\alpha \cdot \mathbf{u})\mathbf{c}_\alpha}{c_0^4} \right] \cdot \mathbf{F_p} \tag{6}$$

Then the source term is included in the mesh concerned by a porous body and are removed in the main flow mesh.

APPLICATION TO FLOW OVER A POROUS SPHERE

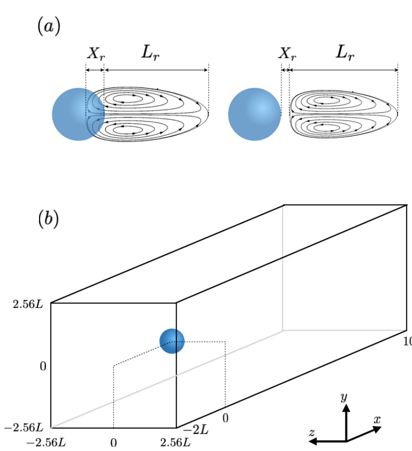

Figure 1: (a) Sketch of the flow configuration depending on the permeability (Da). X_r denotes the penetration or detachment length of the recirculation region and L_r denotes the recirculation length (notation and sketch inspired from [9]). (b) Computational domain.

The test case of the steady and axisymmetric flow past a permeable sphere at $Re = 200$ is here chosen to validate the Brinkman penalization implementation in the LBM and VM frameworks and to compare the two approaches in terms of numerical convergence, precision and ability to capture the correct physics of such flow.

Concerning the expected physics, previous studies (e.g. [9]) demonstrated that, increasing the permeability (i.e. the Da number) of the immersed body, the recirculation region first penetrates inside the body (at the rear) and is then shifted downstream and shrinked until it finally disappears for high permeability values (see Fig. 1(a)).

Based on the very recent work of Ledda [9], used here as a reference, we will validate our methods on the particular case of flow past a permeable sphere at $Re = 200$ with $Da = 10^{-3}$ and $Da = 10^{-2}$, for which we expect to observe the recirculation region entering the rear back of the sphere with $Da = 10^{-3}$ and its disappearing for $Da = 10^{-2}$. The direct numerical simulations will be performed in the physical domain represented in Fig. 1(b) and a grid convergence study will be carried out based on the following meshes:

G1	$80 \times 32 \times 32$
G2	$160 \times 64 \times 64$
G3	$320 \times 128 \times 128$
G4	$640 \times 256 \times 256$
G5	$1024 \times 512 \times 512$

Further simulations at higher Re numbers ($Re \sim \mathcal{O}(10^3)$) for the sphere case will also be proposed for G5 resolution through the two methods in order study more complex physics and to envision the effects of permeability on chaotic wakes.

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